2010-11 academic year

Differential Equations (21602)

Degree/study: Bachelor's degree in Audiovisual Systems Engineering
Year: 1st
Term: 3rd
Number of ECTS credits: 4 credits
Hours of studi dedication: 100 hours
Teaching language or languages: language
Teaching Staff: Edoardo Provenzi, Vanel Lazcano.

1. Presentation of the subject

This subject will provide the students with basic concepts on ordinary differential equations (ODE) and partial differential equations (PDE). Special emphasis will be placed on modelling applications, just to stress the importance (not only theoretical) of these types of equations.

The summarization of the topics taught in the subject is the following:

1. Presentation of differential equation concept: historical notes, definitions, terminology and initial value problems;
2. ODEs and separation of variables: underlying theory and applications. Examples of modelling:

- Population dynamics and logistic models;
- Body free fall and body fall with air resistance;
- Material radiocarbon dating;
- Radioactive decays;
- 'Catenary' as the configuration of balance of a rope supported at its ends;
- Weber-Fechner's law about human sensory perception.

3. Autonomous and nonautonomous first-order linear ODEs: underlying theory and activities. Linearization. Modelling examples:

- RL and RC electric series circuits;
- Newton's law of cooling.

4. Second-order linear ODEs: structural theorems. Homogeneous linear ODEs with constant coefficients: solution method by a related characteristic polynomial. Nonhomogeneous ODEs: similarity method. Modelling examples:

- Spring-mass systems: undamped, damped (overdamped, underdamped and critically damped), driven;
- RLC series circuits;
- Resonance and beats: musical instruments tuning and radio recievers.

5. Power series solutions. Important example: Legendre's equation.
6. Numerical methods to solve ODEs:

- Euler's method(s);
- Heun's method (predictor-corrector);
- Runge-Kutta's method.

7. PDEs: presentation of basic concepts about boundary conditions (Dirichlet or Neumann) and discussion about equations involving standard partial derivatives:

- Wave equation (or Alembert's equation);
- Potential equation (or Laplace's equation);
- Heat-diffusion equation.

Specially, it will be shown the solution technique of "separation of variables" when there is a heat equation with stationary boundary conditions and its relation to Fourier series.

The main aim of the subject is to introduce the students to modelling techniques by the integro-differential calculation the students have learnt in other previous calculation subjects.

These techniques have a nearly universal application field and students will be aware of it during their degree.

2. Competences to be attained

<table>
<tr><th>Transferable skills</th><th>Specific competences</th></tr>
<tr><td>Instrumental

1. Capacity to understand and analyze mathematical formulations.
2. Capacity to identify suitable methodologies to analyze a problem and find a solution.
3. Skill to express accurately mathematical ideas and concepts both orally and written.
4. Capacity to make abstractions.

Interpersonal

5. Capacity to work in groups when finding problem solutions and going into theoretical contents.
6. Capacity to communicate accurate ideas both written and orally.

Systemic

7. Capacity to work autonomously to find a problem solution.
8. Capacity to find the most suitable solutions depending on the aspects of each context.
9. Capacity to infer mathematical notions.
10. Skill to get used to solution checking and interpretation, not forgetting special cases.</td><td>1. Capacity to identify and justify the application of suitable mathematical models to analyze a problem and find its solution.

2. Skill to express accurate mathematical ideas and concepts orally and written.

3. Capacity to understand and know how to reproduce theoretical demonstrations.

4. Capacity to find the solution of the differential equations of the subject.

5. Capacity to model a problem with variation magnitude and speed by a differential equation.

6. Capacity to use the approximation models of the subject to find a solution of differential equations which can not be solved analytically.

7. Skill to know how to recognize the structure of differential equations of partial derivatives and their meaning.</td></tr>
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3. Contents

Unit 1.

Subunit 1.

Presentation of differential equation concept: historical notes, definitions, terminology, initial value problems;

Subunit 2.

ODEs and separation of variables: underlying theory and activities. Examples of modelling:

- Population dynamics and logistic models;
- Body free fall and body fall with air resistance;
- Material radiocarbon dating;
- Radioactive decays;
- 'Catenary' as the configuration of balance of a rope supported at its ends;
- Weber-Fechner's law about human sensory perception.

Unit 2.

Autonomous and nonautonomous first-order linear ODEs: underlying theory and activities. Linearization. Modelling examples:

- RL and RC electric series circuits;
- Newton's law of cooling.

Unit 3.

Subunit 4. Second-order linear ODEs: structural theorems. Homogeneous linear ODEs with constant coefficients: solution method by a related characteristic polynomial. Nonhomogeneous ODEs: similarity method. Modelling examples:

- Spring-mass systems: undamped, damped (overdamped, underdamped and critically damped), driven;
- RLC series circuits;
- Resonance and beats: musical instruments tuning and radio receivers.

Unit 4.

Subunit 5. Power series solutions. Important example: Legendre's equation.

Unit 5.

Subunit 6. Numerical methods to solve ODEs:

- Euler's method(s);
- Heun's method (predictor-corrector);
- Runge-Kutta's method.

Unit 6.

Subunit 6. PDEs: presentation of basic concepts about boundary conditions (Dirichlet or Neumann) and discusión about equations involving standard partial derivatives:

- Wave equation (or Alembert's equation);
- Potential equation (or Laplace's equation);
- Heat-diffusion equation.

Specially, it will be shown the solution technique of "separation of variables" when there is a heat equation with stationary boundary conditions and its relation to Fourier series.

Each unit of theoretical arguments has its revision and consolidation activities.

Finding the solution of these activities will be useful for the students to check their comprehension of the topics of the subject. Time commitment depends on each student.

In seminar sessions, the students should give their solutions of the activities proposed and they also should discuss with the lecturers all the doubts and difficulties when finding solutions of the activities.

Practical sessions will be mainly devoted to modelling problems by means of the differential equations of the theoretical sessions of the subject. Students will be able to appreciate the versatility of differential equations when considering practical problems of physics, engineering, geometrics, biology, electronics, economy and psychophysics.

4. Assessment

The evaluation of this subject consists of two written partial examinations. The first partial examination will take place during first week of May and the second one at the end of the term. The exams consist of activities about the different types of differential equations of the subject. The value of each activity will be communicated before starting the examination.

Important: to pass the subject, the students must get at least a mark of 5 (the maximum is 10) from both partial examinations. If a student doesn't get at least a mark of 5 from any of the two partial examinations, the student will fail the subject and will have to retake the failed partial examination (or both) during retake period in September.

5. Bibliography and teaching resources

- Teaching staff notes.
- D. G. ZILL: *Ecuaciones diferenciales con aplicaciones de modelado*, International Thomson Editores, 1997.
- G. F. SIMMONS: *Ecuaciones diferenciales, Con aplicaciones y notas históricas*, Ed. McGraw Hill, 1993.
- S. G. KRANTZ: *Differential equations demystified*, Ed. McGraw Hill, 2005.
- M. BRAUN: *Ecuaciones diferenciales y sus aplicaciones*, Grupo Editorial Iberoamérica, 1990.
- M. R. SPIEGEL: *Ecuaciones diferenciales aplicadas*, Prentice Hall Hispanoamericana, 1983.
- R. COURANT, F. JOHN: Introducción al cálculo y al análisis matemático Vol. 2, Ed. Limusa, Grupo Noriega Editores, 1999.
- A. QUARTERONI, F. SALERI: *Scientific computing with MATLAB and Octave*, Springer, 2006.