Probability & Statistics (21120)

Degree/study: IBE
Course: second year
Term: first and second term
Number of ECTS credits: 10
Hours of student's dedication: 250
Language or languages of instruction: English
Professor: Christian Brownlees

1. Presentation of the subject

This course is an introduction to probability and its applications. Topics include: basic probability concepts; rules and operations in probability; counting rules; random variables, moments and transformations; discrete and continuous distributions; bivariate distributions; conditional distributions and conditional expectation; the bivariate normal distribution; limit theorems for sums of random variables; applications of limit theorems. There will be sixteen theory lectures devoted to developing concepts and applications, and six practical sessions that will focus on the application of the methods learned in class.

2. Competences to be attained

Specific competences:
1) Acquisition of the basic concepts of probability and statistical inference.
2) Knowledge and understanding of basic statistical calculations and the software tools used for them.
3) The ability to identify the elements making up a univariate statistical model applied to real situations.
4) The ability to use standard statistical packages and to correctly interpret the lists produced.

General competences:
G4. The ability to satisfactorily use the English language for academic purposes (read, write and speak using a medium-high register).
G5. Proficiency in the use of computing tools and their main applications in ordinary academic work.
G9. Consolidated habits as regards self-discipline, personal standards and thoroughness in academic work and in organization and fulfilment of timescales.
G10. A proactive attitude to ascertaining the unknown, essential in all training processes and in all prestigious professional activities.
G11. The ability to apply the knowledge acquired and to adapt it to new situations flexibly and creatively.
G12. The ability to make progress autonomously and continuously in training and learning processes.
G16. Use of the right information in formulating proposals and problem-solving.
G19. Identification of the key factors in a problem.

3. Contents

- Probability: Basic concepts, Laplace rule, counting techniques, conditional probability, independence of events, total probability rule, Bayes's theorem

- Discrete random Variables: Mass probability function, expectation, variance, expectation of a function, moments, properties of the expectation and variance, Chebyshev's theorem.

- Discrete probability models: Uniform discrete, Bernoulli trials, Binominal, Geometric distribution, Poisson distribution, Examples

- Continuous random variables: density function, probability of an interval, distribution function, expectation, variance and moments, percentiles, quantiles, examples.

- Continuous probability models: Uniform distribution, exponential, normal distribution, the 68-95-99.7 rule, distributions associated to the normal (chi-square), applications.

- Bivariate distributions: joint , marginal and conditional distributions (the discrete and the continuous case), independence of random variables, covariance, correlation, the condition expectation (the regression curve), examples

- The bivariate normal distribution, joint, marginals and conditional distributions, the conditional expectation (the linear regression line), applications

- Limit theorems: the concept of independent random variables, law of large numbers, central limit theorem, computing probabilities for the sum of random variables, the normal approximation, applications.

4. Assessment

The assessment will be based on class participation (15%), two intermediate mini tests that will be carried out in the practice sessions (25%) and the final exam (60%).

The minimum mark required to pass the exam is 4.

The assessment of the September exam will be solely based on the written test.

5. Bibliography and teaching resources

5.1. Basic bibliography

- Bertsekas, D. P., and Tsitsiklis, J. N., (2008) Introduction to Probability, 2nd Edition.

5.2. Additional bibliography

- Grinstead, C. M., and Snell, J. L. Introduction to Probability, American Mathematical Society

- DeGroot, M. H. and Schervish, M. J. (2002) Probability and Statistics, (Third Edition), Addison-Wesley

- Baclawski, K. (2008) Introduction to Probability with R, Chapman and Hall/CRC

5.3. Teaching resources

The lectures slides of the course will be available for the students.

6. Methodology

The course is divided in Theory and Practice sessions. Theory sessions will be used to introduce the material of the course. Practice sessions will be used to review problem sets that will be assigned in class.

7. Activities Planning

Topic 1 (2 classes): Random Experiments, Probability Spaces, Conditional Probability, Conditional probability, Independence of events, Total Probability Theorem, Bayes Rules.

Topic 2 (2 classes): Finite Probability Spaces with Uniform Probability, Counting Rules

Topic 3 (2 classes): Random Variables, Discrete Random Variables: Mass Probability Function, Expected Value, Variance, Moments, Properties.

Topic 4 (2 classes): Discrete Random Variables: Bernoulli, Binomial, Poisson, Properties and Applications.

Topic 5 (3 classes): Continuous Random Variables: Gaussian, Exponential, Gamma, Chi Square, Log normal, Properties and Applications.

Topic 6 (3 classes): Multivariate Distributions, the bivariate Gaussian distribution, Properties and Applications

Topic 7 (2 classes): Asymptotics. Sum of independent random variables. The law of large numbers. The central limit theorem. Approximating the distribution of sum of random variables.

- Week 21/09-23/09

Theory: Topic 1 (Thursday & Friday)

- Week 26/09-30/09

Theory: Topic 2 (Thursday & Friday)

Homework: Problem Set 1

- Week 03/10-07/10

Theory: Topic 3 (Thursday & Friday)

Practice: Problem Set 1

Homework: Problem Set 2

- Week 10/10-14/10

Theory: Topic 4 (Thursday & Friday)

Practice: Problem Set 2

- Week 17/10-21/10

Theory: Topic 5 (Thursday)

Practice: First Test based on Problem Sets 1 & 2

Homework: Problem Set 3

- Week 24/10-28/10

Theory: Topic 5 (Thursday)

Practice: Problem Set 3

- Week 31/10-04/11

Theory: Topic 5 & Topic 6 (Thursday & Friday)

Homework: Problem Set 4

Week 07/11-11/11

Theory: Topic 6 (Thursday)

Practice: Problem Set 4

- Week 14/11-18/11

Theory: Topic 6 (Thursday)

Practice: Second Test based on Problem Sets 3 & 4

- Week 21/11-24/11

Theory: Topic 7 (Thursday)

- Week 28/11-02/11

Theory: Topic 7 (Thursday)