Linear Algebra and Discrete Mathematics (21593)

Degree: Bachelor's degree in Computer Engineering, Bachelor's degree in Telecommunications Network Engineering and Bachelor's degree in Audiovisual Systems Engineering
Course: First
Trimester: first and second
Number of ECTS credits: 8
Hours of student dedication: 200
Language or teachinf languages: Catalán and Spanish
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1. Presentation of the Subject

The subject of Linear Algebra and Discrete Mathematics is one of the mathematical fundamental subjects studied within the Bachelor’s Degree in Telecommunications Network Engineering and Bachelor's degree in Audiovisual Systems Engineering. In these degrees, the subject of Linear Algebra and Discrete Mathematics are taught within the first and second trimesters of the First Year being a basic subject worth 8 credits.

This subject stems from the mathematical concepts that students have already worked on in the Baccalaureate Studies developing and widening them. Together with the other subjects related to mathematical basics, it will supply the student with the necessary tools and the mathematical basis with which to work the concepts pertaining to each Degree. In the present Teaching Plan, the competences and abilities to which the learning of the subject will be taught will be detailed, where, parallel to the development and study of the blocks of theoretical contents in which the subject is organized, the practical modules and the associated activities play a fundamental role based on exercises and problems, which aim to consolidate the understanding of the concepts and techniques acquired, which are also complemented by some practices with a computer.

Linear Algebra and Discrete Mathematics is dedicated to an introduction to Linear Algebra and an introduction to Graph Theory and the optimization of flows in networks. It is structured in two major axes. The first is focused on learning: (i) the basic ideas of linear algebra: vector spaces and subspaces, linear independence, dimension, bases, linear applications, determinants, etc., (ii) the solution of linear systems; (iii) values and eigenvectors. The tool or initial idea from which all these competences will be developed is the solution of linear systems by the method of elimination of Gauss.

The second axis is dedicated to developing the essential ideas of Graph Theory, with some notes of combinatorial and optimization algorithms in transport networks. It thus contributes to the learning of important algorithms in the training of an engineer, such as the Dijkstra algorithm or the Ford-Fulkerson algorithm, and also serves as a complement to other subjects, among them the block of algorithmic, programming and Data structures. Learning is based on a practical approach, which includes the study of multiple graph algorithms and computer practices.

In this subject it is intended to provide mathematical training and a greater maturity in the reasoning capacity of the student, thus enhancing their capacity for abstraction. The subject is focused on learning a set of skills and strategies that allow the student to analyze a problem, look for a mathematical model to describe it, solve it and analyze the solution obtained.

2. Prerequisites for the follow-up of the training itinerary

The previous knowledge that this subject presupposes are those of a mathematical base of Baccalaureate level or Vocational Training, in particular about the notions and basic procedures of calculation, geometry and linear algebra. In addition, students are expected to have some familiarity with the arithmetic of complex numbers. As it is (along with Calculus and Numerical Methods) of one of the two subjects of Mathematics of the first and second trimesters of the first course of the three grades, will reinforce the knowledge of students who have deficiencies in elementary mathematics with complementary exercises to achieve a certain homogeneity in the level of all students.

For a good follow-up of the planned training itinerary in this subject, students are expected to understand that learning in this subject is based on their own work and the willingness to learn and understand. The learning of the subject will reinforce its mastery of the scientific language and its abstract reasoning. Students are required to self-demand in reasoning and in the preparation of work, as well as capacity for effort and constructive participation.

3. Competences that must be achieved

Cross-disciplinary competences	Specific competences
Instrumental 1. Ability to understand and analyze mathematical statements. 2. Ability to identify the appropriate methodology to analyze a problem and find the appropriate solution. 3. Ability to express ideas and mathematical concepts orally and in writing with precision. 4. Abstraction capacity. 5. Systematization capacity. *Systemic* 1. Capacidad de trabajar autónomamente en la resolución de problemas. 2. Capacidad de aprender de los errores propios y de los de los demás. 3. Capacidad de buscar la solución más adecuada en función de las características de cada problema/situación/contexto. Capacidad de inferir nociones matemáticas.	*Linear Algebra Axis* 1. Understand the algebra and the basic geometry of complex numbers. 2. Master the concepts of vector and matrix and operations with vectors and matrices. 3. Understand the geometry of systems of linear equations. 4. Understanding and mastering the Gaussian elimination method for the resolution of systems of linear equations. 5. Understand the basic concept of a vector space. 6. Understanding of the four fundamental vector subspaces. 7. Understanding of the concept and technique of the base changes. 8. Understand and master the concept of orthonormalization of a base, and in particular of the Gram-Schmidt method. 9. Understanding of rotations and basic transformations. 9. Domain of the concept of value and own vector. 10. Understand the concept of diagonalizable matrix and the case of symmetric matrices. *Graph Theory axis and network optimization* 11. Become familiar with the elements of Graph Theory and understand them. 12. Master the different graph models, and the concept of multigraph. 13. Acquire the concept of graph isomorphisms. 14. Study of the characteristic properties of graphs, such as the principle of shocked hands. 15. Analyze the different representations of graphs in a computer. 16. Understanding of the concept of Tree. 17. Study of different search algorithms in trees. 18. Mastery of the concept of generator tree and algorithms to make calculations of this type. 19. Study and mastery of the characterization of the Eulerian Graphs. 20. Study of Hamiltonian graphs. 21. Knowledge of the algorithms to obtain a Eulerian circuit and a Hamiltonian cycle.

	22. Study of minimum cost roads. 23. Study of the traveler's problem. 24. Understand the coloring of graphs. 25. Understanding and analysis of the graphs case in the communications networks. 26.- Understanding of the concept of connection for directed and non-directed graphs and their applications. 27. Understanding the optimization of flows in networks. 28. Know and know how to implement the Ford-Fulkerson algorithm to solve the problem of maximum flow in a network.

4. Contents

4.1. Content Blocks

The subject is structured in four blocks of contents for the Linear Algebra axis and five content blocks for the axis of Graph Theory and Network Optimization:

Axis I: Linear Algebra

- Content block 1. Resolution of systems of linear equations.
- Content block 2. Vector spaces and their subspaces.
- Content block 3. Euclidean vector spaces.
- Content block 4. Diagonalization.

Axis II: Graph Theory and Network Optimization

- Content block 5. Graph theory elements.
- Content block 6. Trees
- Content block 7. Eulerian and Hamiltonian graphs.
- Content block 8. Roads and minimum cost cycles.
- Content block 9. Optimization of flows in networks.

4.2. Organization and specification of contents

Content Block 1. Resolution of Systems of linear equations.

Concepts	Procedures
1. Complex numbers 2. Functions. Inversas. Graph of a function. 3. Vectors and Matrices. 4. Linear equations. Systems 5. The geometry of systems of linear equations. 6. Singular systems. 7. The decomposition LU	- Basic algebraic and geometric properties of the complexes. - Differentiation of the different types of functions. -Demonstration by Reductio ad absurdum -Operations with vectors and matrices -The Gaussian method. - LU method. -Calculation of the inverse matrix.

Content Block 2. Vector spaces and their subspaces

Concepts	Procedures
1. Vector subspaces 2. The core and the image of a linear applicationA 3. The four fundamental subspaces. 4. Linear independence Bases. Dimension.	- Calculation of the four fundamental subspaces of A

Content Block 3. Euclidean vector spaces

Concepts	Procedures
1. The Determinant 2. Base Changes 3. Orthogonalization 4. Rotations. Orthogonal matrices.	- Calculation of determinants. - Base changes. - The characteristic polynomial. - The Gram-Schmidt method.

Content Block 4. Diagonalization

Concepts	Procedures
1. Values and eigenvectors 2. Diagonalization 3. Symmetric matrices.	- Calculation of values and eigenvectors - Diagonalization procedure

Content Block 5. Graph theory elements

Concepts	Procedures
1. Graphs and multigraphs. 2. Graph isomorphism. Subgraphs. Complete graphs. Bipartite graphs. 3. The principle of shocked hands. 4. Planar graphs. Euler's theorem. Kuratowski's theorem. 5 Representation of a graph by computer.	- Strategies of using characterization, properties and equalities on graphs to deduce other properties of them. - The circle-string method. - Demonstrations by induction.- Demonstrations by Reductio ad absurdum.

Content Block 6. Trees

Concepts	Procedures
1. Definitions and basic results. Characterizations. Trees with roots. M-ary trees. Balanced trees	- Characterizations. - Algorithms of depth and extension searches

2. Searches on trees. In depth and extension searches.	- Algorithms of Kruskal and Prim.
3. Generator trees Generating trees of minimum cost.	

Content Block 7. Eulerian and Hamiltonian graphs

Concepts	Procedures
1. Eulerian circuits and routes. 2. Cycles and Hamiltonian roads. 3. The problem of the traveler.	- Fleury algorithm for obtaining a Eulerian circuit. - Robert and Flores Algorithm for obtaining a Hamiltonian cycle. - The Branch and Bound method.

Content Block 8. Roads and Minimum Cost Cycles

Concepts	Procedures
1. Minimum cost paths. 2. Coloring of graphs. 3. The problem of the surveillance of an art gallery.	- Algorithm of Dijkstra. - Floyd Algorithm. - Obtaining maximum sets of independent vertices. Algorithm of Bron and Kerbosch.

Content Block 9. Optimization of Network Flows

Concepts	Procedures
1. Transport and communications networks. 2.- k-Connectivity for vertices and edges. 3.- Results for directed graphs. 4. Optimization of Flows in Networks.	- Cuts for vertices and for edges in networks. Minimum flows. - Optimization algorithms. - Max-Flow Min-Cut. Algorithm of Ford and Fulkerson

5. Evaluation of the level of acquisition of competences

During the course of the course a continuous evaluation will be done through the learning activities that are proposed. It is intended with this an effective informative feedback for the students and that it is an aid to the verification of the acquisition of the different competences. The objective is to detect difficulties in time and provide feedback to students to guide their learning and introduce the necessary modifications.

These evaluation mechanisms, all of them mandatory in the continuous assessment, are structured in two main axes, according to the learning structure of the subject. They are the ones that follow:

5.1. Linear Algebra Axis

- Theory controls: they will be two controls of short questions or test type, whose objective is to follow up on the concepts explained in the theory classes. Each theory control counts 10% of the Lineal Algebra axis qualification. The first theory control will be on content blocks 1 and 2, and the second, on content blocks 3 and 4.
- Problems controls: they will be two controls of exercises or problems, analogous or of analogous difficulty to those that are made from the collections of exercises or problems in the practice sessions and / or seminar. Each problem control account for 20% of the Lineal Algebra axis qualification. The first control of problems will be on blocks of contents 1 and 2, and the second, control on blocks of contents 3 and 4.
- Partial Exam: it is a test on the four blocks of contents 1, 2, 3 and 4, that is, especially the Linear Algebra axis. It will consist of exercises and representative problems where all the concepts of theory will have to be applied and demonstrate a mastery of the competences developed in the Linear Algebra axis. It counts 40% of the Lineal Algebra axis qualification.

Therefore, the note corresponding to the Linear Algebra axis is calculated as follows:
10% (1st control theory) + 10% (2nd control theory) + 20% (1st control problems) + 20% (2nd control problems) + 40% (exam note) = Note Algebra Linear axis
This note must be at least 5 to approve the Linear Algebra axis.

5.2. Graph Theory and Network Optimization Axis

The evaluation mechanisms of this block, all of them obligatory in the continuous evaluation, are:

The accomplishment of 4 computer practices in the sessions of problems that will require the realization and delivery of a previous study. The reports of the practices will have to be delivered at the end of the corresponding session. The practical note will be weighted by the number of practices that are attended in person. The final note of practices counts 25% of the grade of Graph Theory and Network Optimization axis.

Controls of problems: they will be two controls of exercises or problems, analogous or of difficulty analogous to those that are made from the collections of exercises or problems in the sessions of practices and / or seminar. Each problem control account for 12.5% of the grade of Graph Theory and Network Optimization. The third control of problems of ALMD will be on the blocks of contents 5, 6 and 7, and the fourth, on the blocks of contents 8 and 9.

- Theory controls: they will be two controls of short questions or test type; whose objective is to follow up on the concepts explained in the theory classes. Each theory control account for 7.5% of the grade of Graph Theory and Network Optimization. The third ALMD theory control will be on the content blocks 5, 6 and 7, and the fourth, on the content blocks 8 and 9.

- Midterm Exam: this is a test on the four content blocks 5, 6, 7, 8 and 9, that is, on the axis of Graph Theory and Network Optimization. It will consist of exercises and representative problems where all the concepts of theory will have to be applied and demonstrate a mastery of the competences developed in the axis of Graph Theory and Network Optimization. Account for 35% of the Grade Theory and Network Optimization axis qualification.

Therefore, the note corresponding to the axis of Graph Theory and Network Optimization is calculated as follows:
25% (internship note) + 7.5% (3rd control theory) + 7.5% (4th control theory) + 12.5% (3rd control problems) + 12.5% (4th control problems) + 35% (note examination) = note axis of Graph Theory and Optimization in Networks

This grade must be at least 5 to pass the axis of Graph Theory and Network Optimization.

The Final Note of the subject Linear Algebra and Discrete Mathematics is obtained by calculating the average of:
- the note of the axis of linear algebra, whenever it is greater than or equal to 5,
- the graph axis mark, provided it is greater than or equal to 5

In addition, the following criteria are taken into account:
- To pass the subject you must obtain a Final Score greater than or equal to 5 (five).
- If the mark of the linear algebra axis was less than 5, in the exam period of the 2nd quarter (ordinary call in March) the student can also examine all the contents of the linear algebra axis (blocks 1, 2, 3 and 4)).

- In the event that he suspends or fails to attend the examination of the ordinary session in March, the student may submit to the September session and perform a final exam (all content blocks) of both axes. If one of the two axes has been approved, it will only be necessary to examine the suspended axle.

6. Bibliography and teaching resources

6.1. Basic bibliography (paper and electronic support)

- G. STRANG, Linear Algebra and its Applications, Harcourt Brace Jovanovich International Edition, 1986. (también: http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2005/)
- G. STRANG, 18.06 Linear Algebra Course, MIT Open Courseware, http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2005/index.htm
- F. CEDO i V. GISIN, Álgebra Básica, Manuales de la UAB, 1997.
- I.V. PROSKURIAKOV, 2000 Problemas de Álgebra Lineal, Ed. Reverté, 1991.
- J.R. EVANS, E. MINIEKA, Optimization Algorithms for Networks and Graphs, Marcel Dekker, 1992.
- R. BHARATH, Computers and Graph Theory,Ellis Horwood, 1991
- J. FÀBREGA, Teoria de Grafs,Edicions de la UPC, 1997.
- J.A. BONDY & U.S.R. MURTY, Graph Theory with Applications (http://www.ecp6.jussieu.fr/pageperso/bondy/books/gtwa/gtwa.html)

6.2. Complementary bibliography (paper and electronic support)

- M. CASTELLET i I. LLERENA, Álgebra Lineal y Geometria, Manuales de la UAB, 1990.
- F.R. GANTMACHER, Théorie des Matrices,Editions J. Gabay, 1990.
- P. HALMOS, Finite-Dimensional Vector Spaces,Springer Verlag.
- AUBANELL, A. BENSENY y A. DELSHAMS, Útiles Básicos de Cálculo Numérico, Ed. Labor, 1993.
- W. K. NICHOLSON, Álgebra Lineal con aplicaciones, Mc Graw Hill, 2003.
- J.M. BASART i MUÑOZ, Grafs: Fundamentos y Algorismos, Manuales de la UAB, 13, 1994.
- N.L. BIGGS, Discrete Mathematics
- K.H. ROSEN, Discrete Mathematics and its Applications
- M. BRUNAT BLAY, Combinatoria y Teoria de Grafs, Ediciones de la UPC.
- X. FRANCH GUTIÉRREZ, Estructuras de Datos, Ediciones UPC, 1994.
- J. GIMBERT, R. MORENO, J.M. RIBÓ i M. VALLS, Acercamiento a la Teoria de Grafs y a sus Algorismos, EINES 23, 1998.
- TUCKER, Applied Combinatorics, Wiley, 1995.

6.3. Teaching material of the subject

- To each session of theory will correspond a teaching material that the teacher will deliver to the student through the Global Classroom.
- For each session of problems there will be a collection of problems that the teacher will deliver to the student through the Global Classroom before the realization of the practice.

7. Methodology

In the first year of the degree there are two / four theory groups. These theory groups are divided into 3 groups of practices. In turn, each group of practices is divided into two seminars, each with half of the students.

The fact of differentiating between three types of sessions will allow us to strengthen and evaluate the various competencies that we intend to achieve throughout the course. It should be emphasized that the seminar sessions strongly favor the acquisition of transversal competences.

7.1. Plenary sessions

It consists of eighteen two-hour sessions attended by the entire group. The teacher carries the weight of the session since it is dedicated to explaining on the blackboard the theoretical concepts of the subject so that later they can be put into practice. The teacher will be responsible for proposing and solving model examples so that the theory is clear and for students to have a first approximation to what will be treated in the session of problems.

Axis I: Linear Algebra
· Session 1: Presentation of the subject. Complex numbers. Functions
· Session 2: Vectors and matrices. Linear applications
· Session 3: Geometry of systems of linear equations. Resolution of systems of linear equations. Gaussian method.
· Session 4: Decomposition LU. Calculation of the inverse matrix.
· Session 5: Vector spaces and their subspaces. The four fundamental subspaces. Linear independence Bases.
· Session 6: The four fundamental subspaces (continued).
· Session 7: Determinants, applications. Base changes
· Session 8: Eucllidian vector spaces. Orthogonalization The Gram-Schmidt method. Orthogonal matrices, rotations.
· Session 9: Values and eigenvectors. Diagonalization.
. Session 10: Diagonalization. Symmetric matrices.

Axis II: Graph Theory and Network Optimization
· Session 11: Introduction. Graphs and subgraphs. Graph models. Basic properties
· Session 12: Isomorphisms. Planar graphs and dual graphs. Representation of graphs by computer.
· Session 13: Trees. Searches.
· Session 14: Minimum cost trees. Connectivity
· Session 15: Eulerian graphs. Hamiltonian graphs.
· Session 16: Roads and minimum cost cycles.
· Session 17: Coatings and Graphing.
· Session 18: Transport and communications networks.
. Session 19: Optimization of Network Flows.

7.2. Problem Sessions

There are sixteen sessions of one hour, organized in groups of thirty students, in which the professor of practices proposes a series of problems to be carried out of a collection that the students will have prepared since they will previously have access to them. The general dynamics of these sessions is the following: first, the teacher performs a typical exercise to remember the theoretical concepts that are applied and to give a resolution method to follow. In the successive problems, it is up to the students to solve the problems and go to the blackboard to explain their classmates how they have done it. Both the students and the teacher have to verify that the problem has been solved well and can propose questions.

Axis I: Linear Algebra
· Session 1: Complex numbers. Functions Vectors and matrices. Interpretation of matrices as a particular case of linear applications.
· Session 2: Resolution of systems of linear equations. Gaussian method. Decomposition LU.
· Session 3: Calculation of the four fundamental subspaces.
· Session 4: 1st problem control.
· Session 5: Determinants. Base changes
· Session 6: Orthogonalization. The Gram-Schmidt method.
· Session 7: Values and eigenvectors. Orthonormal matrices, rotations.
. Session 8: 2nd control of problems.

Axis II: Graph Theory and Network Optimization

In this block, some sessions will be done with the help of a computer. For the realization of these, the student must perform a prior study before the session. During the session, the student must be able to analyze the results obtained through the simulations and, upon completion, the student must submit a report to the teacher.

· Session 9: Problems at the beginning of shocked hands, isomorphisms in graphs and the circle-string method.
· Session 10: Computer practice on trees and generators.
· Session 11: Problems on Eulerian and Hamiltonian graphs.
· Session 12: Computer practice on roads and minimum cost cycles.
· Session 13: Computer practice on graph coloring.
. Session 14: Computer practice on optimizing flows in networks.

7.3. Seminary sessions

It consists of twenty one-hour sessions in small groups (maximum fifteen students). In these sessions different types of activities are carried out guided by the seminar teacher.

Axis I: Linear algebra
· Session 1: Applications, practical examples and problems about the geometric interpretation of systems of

linear equations, which are solved jointly. Resolution of questions posed by the students themselves.
· Session 2: Calculation of the inverse matrix.
· Session 3: Linear independence and range of a system. Bases.
· Session 4: Calculation of the four fundamental subspaces. Bases. Resolution of questions posed by the students themselves.
· Session 5: 1st theory control.
· Session 6: Basic changes.
· Session 7: Gram-Schmidt, orthogonalization.
· Session 8: Orthogonal matrices, rotations.
· Session 9: Diagonalization. Symmetric matrices.
. Session 10: 2nd Control of theory.

Axis II: Graph Theory and Network Optimization
· Session 11: Applications, practical examples and problems solved jointly of the sessions of theory 10 and 11. Resolution of questions posed by the students themselves.
· Session 12: Applications, practical examples and jointly solved problems of trees, generators, and connectivity ...
· Session 13: Applications, practical examples and solved problems of the Eulerian and Hamiltonian theme.
· Session 14: Applications, practical examples and solved problems of the topic of roads and minimum cost cycles. The Traveling Salesman Problem Branch & Bound Algorithm.
· Session 15: Control of theory.
· Session 16: Applications, practical examples and problems solved jointly on the blackboard about graph coloring.
· Session 17: Applications, practical examples and solved problems of graph theory to communication networks.
· Session 18: Applications, practical examples and problems solved jointly by the optimization theory of flows in networks.
· Session 19: General review of all the concepts that have been seen in class. Resolution of questions posed by the students themselves.
. Session 20: Control of theory.